REVISION OF STATE OF THE ART CONTINGENT LIABILITY MANAGEMENT
Bibliography

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1. General Framework

The intention of this paper is to provide an overview of the state of the art literature on contingent liability management. First, it describes the risks entailed by infrastructure projects and defines the notion of contingent liabilities and their impact in public fiscal stability. Accordingly, it brings forth the set of guidelines proposed by the International Monetary Fund on fiscal transparency, and consequently, on contingent liability management, to be ideally followed by international governments. Second, it describes techniques currently in use by the private sector to manage its holdings of contingent liabilities, stressing the Basle Agreement and the Value at Risk analysis as potential instruments to be likely adapted by national governments. It also outlines approaches specifically suggested for public management of these liabilities, underlining the valuation of government risk exposure by means of VAR and stochastic methods, and describes the risk exposure valuation experience of the Colombian government. Finally, given the preeminent place that guarantees hold among contingent liabilities, a review of guarantee pricing methods is offered, underlining Option Pricing Theory and Defaultable Bond Valuation, and providing a pricing illustration using Black and Scholes formulation.

Risks Entailed by Infrastructure Projects

The private sector is often reluctant to bear some of the risks implied by the nature of infrastructure projects. Due to the prolonged constructions periods entailed by these projects, investors in infrastructure generally confide their financial resources to host-countries over very long time periods, even decades. Unlike portfolio investments, investors lack the option to promptly withdraw their resources given the case of political instability or economic volatility.

Thus, infrastructure investors must cope not only with business risks that would be considered normal for any investment, such as commercial and financial risks, but also with risks that might be under the direct control or influence of the governments of host-countries, and directly associated with policies undertaken over extended periods of time.[1] These policies may over time affect demand, payments, foreign exchange rates, interest rates, currency transferability, and the overall political and regulatory environment, thus menacing expected revenues, and even the investments themselves, in the form of expropriations, breach of contracts, nullification of permits, abrogation or amendment of existing regulations, sudden devaluations or changes in interest rates, modifications in trade regimes, wars and sabotages, among others.[2]

The perceived existence and degree of these risks determine the risk assigned to a given country by investors, and thus the rate of return required by them in order to allocate their funds in that country. Country risk assessments are commonly offered to international market participants by credit rating agencies such as Standard & Poor's ™, Moody's™, and Duph and Phelps™. Generally, investors are willing to allot their funds at low cost of capital levels in countries with low country-risk, that is, with macroeconomic and political stability, and strong and credible regulatory frameworks. In contrast, in cases of high

[1] Klein, (1997). P. 1
[2] Irwin et al. (1997). P. 4

country risk, the cost of capital will be substantially elevated, and it may even be difficult to attract investments at all.

Contingent Liabilities

In cases of high country risk, contingent liability instruments have been widely employed by governments to attract private capital flows in infrastructure projects at satisfactory rates to support infrastructure projects. By means of these instruments, governments share with the private sector some or all of the risks listed before, thus lowering investor's required rate of return.

Contingency is defined by the International Accounting Standards Committee as a situation or condition whose ultimate outcome is determined only by the occurrence, or non-occurrence, of one or more current events.[3] Hence, the government support committed through a contingent liability instrument, is provided only in the presence of an event determined ex-ante. Given such case, the commitment becomes a direct liability for the government.

The most common form of contingent liabilities are guarantees. Yet other forms of contingent support instruments are: State insurance schemes for deposits or minimum returns on pension funds, bailout of subnational entities or financial system, military spending in cases of war, and disaster relief, among others.

The flexibility of contingent liability instruments, specifically guarantees, offer some advantages to governments: as indicated before, they might drive down the cost of capital for a project in a given country; they help reinforce commitments of government agencies and subnational entities; they can be shaped to cope with a wide assortment of risks that are not subject to project or country limits[4] and, they do not represent immediate cash outlays.[5]

On the other hand[6], contingent liabilities pose potential fiscal risks to governments and create moral hazard in the markets. They are often not officially reported, in other words, they are not directly associated to any existing budgetary program, which obstructs their monitoring and control. Since they are not current outlays and they are not reflected in the budget, contingent liabilities allow politicians to pursue policy goals that are not necessarily budgeted and may also compel governments to delay structural reforms. This situation can encourage short-term minded policymakers to provide concealed government support to determined projects and interest groups, while simultaneously accumulating excessive contingent commitments whose fiscal costs may not be disclosed for an undetermined period of time until a discrete event activates the claims against the government's resources. Such behavior generates uncertainty about future public financing requirements and endanger future fiscal stability.[7]

[3] Quoted in Polackova (1998), P. 2.
[4] Smith (1997). P. 71
[5] Lewis and Mody (1997). P. 133
[6] Polackova, op cit. P. 5
[7] Ibidem. P. 11

Fiscal Transparency Code

The International Monetary Fund has elaborated a comprehensive set of guidelines conducing to a sound control of contingent liabilities on the part of national governments. In particular, the IMF Code of Good Practices on Fiscal Transparency provides a series of steps oriented towards enhancing the accuracy of fiscal planning and reporting in the presence of contingent liabilities, thus improving credibility in fiscal policy through public disclosure and accountability of government risk exposures to ultimately ensure macroeconomic stability and economic growth.

In this respect, the Code asserts that "Statements should be published with the annual budget giving a description of the nature and fiscal significance of contingent liabilities, tax expenditures, and quasi-fiscal activities".[8]

Accordingly, section 2.1.3 of the corresponding Manual on Fiscal Transparency offers relevant examples of contingent liabilities, both explicit and implicit, warns against the risks that they might entail and the shortcomings of traditional cash flow based budget accounting. The Manual further describes the advantages of accurate reporting of contingent liabilities in the management of fiscal risks and in the design of policies that involve risk-sharing between the government and the private sector. It recommends as well the inclusion of a statement of the main central government contingent liabilities, and that it be reported as part of a broader fiscal risk assessment. It is suggested that such statement should include a brief indication of the nature of each contingent liability and the beneficiaries, both to enable adequate assessment of "potential fiscal significance and to reduce the possibility of abuses through preferential treatment". Finally, it advises: a) the inclusion of an estimate of the expected cost of each contingent liability (up to the limit set by technical possibilities); b) the provision of information about the basis on which the estimates have been calculated and, c) the registry of those contingent liabilities reported in the previous year's budget resulting in expenditure during the current year.

To the extent that these guidelines may be gradually adopted by IMF members, and thus might eventually serve as a benchmark of sound fiscal policymaking, it is advisable for national governments to take into consideration the measures proposed by the Code, in particular regarding contingent liability management.

Contingent Liability Management Methods Employed by the Private Sector

The private sector, specifically corporations, commercial banks and insurance companies, has made substantial progress in dealing with contingent liabilities in the past decade.[9] Along this line, Schick suggests that since the business sector counts with more advanced statistical tools and hedging strategies for risk assessment, measurement and management, there is no reason for governments not to adapt relevant commercial practices to their fiscal risk analysis and supervision.[10] In this respect, two examples of contingent liability management and determination of capital adequacy in commercial banking that could be illustrative to the public sector are offered: Basle Agreement Capital Adequacy Ratio and Value at Risk VAR.

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[8] Code of Good Practices on Fiscal Transparency. Section 2.1.3.
[9] Polackova, op cit. P. 5
[10] Schick (2000), P. 10

Basle Agreement Capital Adequacy Ratio

After several international cases of commercial bank overexposure to different kinds of risk, the Basle Committee on Banking Supervision of the Bank of International Settlements, in conjunction with the financial authorities of member countries, devised in 1988 a risk-based capital ratio for commercial banks to be enforced by national financial regulators within their jurisdictions. This capital ratio has become known as the Basle Agreement, and it allows to distinguish between adequately capitalized banks and undercapitalized banks, thus signaling the conditions under which corrective actions should be taken by financial authorities to prevent overexposures to risks in the financial system.

In accordance with the Basle Agreement, national financial authorities have implemented local regulations governing capital adequacy of banks, which measure a bank's capital in regards to its risk-adjusted asset holdings and risk-adjusted off-balance sheet items. The Federal Reserve System of the United States implemented in the past decade the Credit-Conversion Factors for Off-Balance Sheet Items for State Member Banks. These credit conversion factors are applied to the nominal principal amount of a bank's exposure to off-balance sheet items in order to generate a credit equivalent amount that in turn is weighted according to the credit rating category of the counterparty.

It should be noted that off-balance sheet items (OBI's) in commercial banking are analogous to direct explicit liabilities: they are obligations of banks that may or may not be called, such as bank guarantees; they are liabilities that depend on some action or event for the bank to pay them.[11]

Conversion Factor	OBI Contingent Contract
100%	• Sale and repurchase agreements and assets sold with recourse that are not included in the balance sheet; • Direct credit substitute standby letters of credit • Forward agreements to purchase assets
50%	• Performance-related standby letters of credit • Unused portion of loan commitments with original maturity of more than a year
20%	• Commercial letters of credit • Bankers acceptances conveyed
10%	• Unused portions of commitments with an original maturity of one year or less, or which are cancelable at any time

Sources: Scott and Wellons. P. 231, and Saunders, P. 459

The rationale supporting each conversion factor is that "the more an item is likely to be called and the greater the risk that the counterparty will be unable to pay, the closer it is to an asset and the higher should be the conversion factor".[12] After multiplying the nominal

[11] Scott and Wellons (2000). P. 231.
[12] Ibidem. P. 232

value of each OBI by the respective credit conversion factor, the result is then weighted down as summarized below, according to the credit rating risk of the counterparty.

OBI Face Value x Conversion Factor = Credit Equivalent Amount x Risk Weight = Weighted-Risk OBI

In this case, the smaller the risk weight, the greater the creditworthiness of the counterparty, as the following table indicates.

Weight	Risk Categories
0%	• Cash • Federal Reserve Bank balances • Securities from OECD governments and some US agencies
20%	• Cash items in the process of collection • OECD interbank deposits and guaranteed claims • Some non-OECD bank and government deposits and securities • General obligation municipal bonds • Some mortgage backed securities • Claims collateralized by the US treasury and some other government securities • Claims on, or guaranteed by official multilateral lending institutions in which the US is a shareholder or contributing member
50%	• Revenue bonds or similar obligations that are obligations of public sector entities in OECD countries • Assets secured by a first mortgage on a one-four family residential property that are not more than 90 days past due • Other revenue municipal bonds
100%	• All other claims on private obligors • Claims on non-OECD financial institutions with residual maturity exceeding one year • Obligations issued by state and local governments repayable solely by a private enterprise • Capital instruments issued by other banking organizations • All other on-balance sheet assets not listed in the categories above

Sources: Sinkey, P. 605 and Saunders, P. 458

The resulting Weighted-Risk OBI is incorporated in the denominator of the Basle Capital Adequacy Ratio, which rules that a bank's capital must be equal or greater than 8% of its risk-adjusted assets:

$$\frac{Tier1 + Tier2}{(RiskWeightedAssets + RiskWeightedOBI's)} \geq 8\% \qquad (1)$$

Where a bank's capital resources are split between Tier 1 (equity and disclosed reserves) and Tier 2 (undisclosed reserves, asset revaluation reserves, general provisions, hybrid debt/equity instruments and subordinated term debt).[13]

In the case where banks to not fulfill the required capital adequacy ratio, they become subject to limits on their lending capacity, and to penalties that may take the form of increments in the reserves commercial banks must place in central banks and in the insurance fees that banks must pay to national deposit insurance schemes.

[13] Ibidem. P. 217

In sum, the Basle Capital Adequacy Ratio allows national financial authorities to comprehensively monitor and control the risk-exposure of the local financial system, thus facilitating a prompt and proper implementation of corrective measures to protect depositor's savings and the overall national economic environment. Its main draw back is that it focuses almost exclusively in credit risk, ignoring other factors such as market risk and diversification effects.

Sample of Risk-Based Capital Ratio

The following tables are provided to illustrate the calculation of the Capital Ratio for an hypothetical commercial bank in accordance to the Federal Deposit Insurance Corporation Act of 1991 of the United States, which complies with the guidelines set by the Basle Agreement.

Assuming the following Balance Sheet and Off-Balance Sheet Items for an hypothetical commercial bank with capital equal to $9,000:

Balance sheet assets		
Cash	$	15,000
US Treasuries	$	13,000
Balances at domestic Banks	$	6,000
General obligation municipal bonds	$	8,000
Loans to private corporations	$	75,000
Total Balance Sheet Items	$	100,000
Shareholders equity	$	7,000
Retained Earnings	$	2,000
Total Capital	$	9,000
Standby letters of credit	$	15,000
Long term legally binding commitments to private corporations	$	13,000
Commercial letters of credit	$	9,000
Total Off-Balance Sheet items	$	45,000

The Capital Adequacy Ratio would be calculated by first computing the credit equivalents of the Off-Balance Sheet Items according to the corresponding conversion factors listed in the previous section:

Credit Equivalent Conversion of Off Balance Sheet Items

	Face Value	**Conversion Factor**	**Credit Equivalent**
Standby letters of credit	$ 15,000	1	$ 15,000
Long term legally binding commitments to private corporations	$ 13,000	0.5	$ 6,500
Commercial letters of credit	$ 9,000	0.2	$ 1,800

And then by multiplying each asset, including the credit equivalent of the OBI's estimated above, by the respective risk weight as described in the earlier section:

0% Risk Category	Face Value	Conversion Factor	Risk Weighted Value
Cash	$ 15,000	0	$ -
US Treasuries	$ 13,000	0	$ -
20% Risk Category			
Balances at domestic Banks	$ 6,000	0.2	$ 1,200
Credit equivalent of Standby letters of credit	$ 15,000	1	$ 15,000
50% Risk Category			
General obligation municipal bonds	$ 8,000	0.5	$ 4,000
100% Risk Category			
Loans to private corporations	$ 75,000	1	$ 75,000
Credit equivalent of long term legally binding commitments to private corporations	$ 13,000	0.5	$ 6,500
Credit equivalent of commercial letters of credit	$ 9,000	0.2	$ 1,800
Total	**$ 154,000**		**$ 103,500**
Risk Based Capital Ratio			**8.7%**

So $9,000/$103,500 = 8.7%. This hypothetical commercial bank would comply with Basle's Capital Adequacy Ratio.

Value at Risk (VAR)

Parallel to the Basle Agreement, commercial banks had traditionally developed their own internal models for assessing their own capital adequacy and estimate the probability that actual loan losses will exceed loan provisions[14] while accounting for market risk. One of these approaches was Value at Risk method, which was developed to "measure the worst expected loss over a given time interval under normal market conditions at a given confidence level".[15] Banks have used this method to determine the value at risk in its loan portfolio by predicting the probability of default, the level of capital to cover that risk, and the likely recovery of assets after a default.[16] More specifically, VAR estimates how much of a bank's portfolio could be lost in a given time period due to adverse movements in financial prices with a one to five percent probability of occurrence. If a bank reports its

[14] Ibidem, P. 246
[15] Jorion (1997), P. xiii
[16] Scott and Wellons, op cit. P. 245

daily VAR at $1 million at a 99% confidence level, it is saying that under normal market conditions, there is only a chance in 100 of losses being rather than $1 million.[17]

The VAR model has been criticized[18] for assuming that correlation history of asset prices will hold in successive time periods, for unrealistically assuming normal distribution of prices, and for not being considered appropriate for assets and liabilities that are not publicly traded since extensive history data is not available (the latter shortcoming is overcame by means of Monte Carlo simulation). Nevertheless, a significant number commercial banks worldwide currently use this approach to determine the market risk exposure of its asset holdings (loans included, despite their lack of trading history) and consequently, their capital reserves levels. The summary of total market risk exposure of an institution in a single number explains why financial institutions, financial regulators, and even non-financial corporations, widely employ VAR as a means to facilitate a straightforward risk exposure reporting to the general public, as an instrument for optimal resource allocation, and as a benchmark to evaluate performance in regards to exposure to risk.

VAR is derived from probability distributions that can be drawn either empirically or by approximating the distribution by a normal curve, in which case VAR is derived from the standard deviation.[19]

Jorion (1997) describes the steps to calculate VAR as follows: first, both a time horizon and a confidence level must be set (the Basle Committee rules the utilization of a 10 day period and a 99 percent confidence level; otherwise a 95 percent is also acceptable). Then, the value of a portfolio at the end of the chosen time horizon is defined as $W= W_0 (1+R)$, where W_0 is the initial investment and R is the rate of return, whose expected return and volatility are μ and σ, respectively. Defining the lowest portfolio value at the given confidence level c as $W^*= W_0 (1+R^*)$ allows to define VAR as the dollar loss relative to the mean:

$$VAR\mu = E(W) - W^* = -W_0(R^* - \mu) \qquad (1)$$

Estimation of VAR is equivalent to finding the minimum value of W^*, or the cutoff return R^*. In its most general form, VAR may be derived from the probability distribution of the future portfolio value f(w). At a given confidence level c, we seek to determine the worst possible realization W^* such that the probability of a value lower than W^*, $p= P(w \leq W^*)$ is 1 - c:

$$1 - c = \int_{-\infty}^{W^*} f(w)dw = P(w \leq W^*) = p \qquad (2)$$

Which means that the area from $-\infty$ to W^* must sum to p = 1 – c, or 1 percent if Basle Committee rules were followed. As an illustration, let us suppose that we have 254 independently distributed observations of daily revenues for a institution, with a mean of $5 million, VAR would be computed at the 95% confidence level by finding W^* such that the number of observations below the value of W^* is 254 x 5% = 12.7. Assuming that 11

[17] Jorion, op cit. P. xiii
[18] The Economist, February 28, 1998, p. 80
[19] Jorion, op cit. P. 87

observations are below -\$10 million, and that 15 observations are below -\$9 million, through interpolation W^* can be estimated as -\$9.6 million. Thus, the VAR for the daily revenues measured relative to the mean would be:

$$VAR = E(W) - W^* = \$5.1 \text{ million} - (-\$9.6 \text{ million}) = \$14.7 \text{ million}$$

In terms of absolute dollar loss, VAR(zero)= $W_0 - W^*$ = 0 - \$9.6 million = - \$9.6 million.

It should be noted that the standard deviation was not employed to calculate VAR. In this respect, Jorion indicates that by assuming the distribution to be normal, VAR computation can be derived from the standard deviation by applying a multiplicative factor dependent on the confidence level. This can be done by transforming the general distribution f(w) into a standard normal distribution $\Phi(\varepsilon)$, where ε has a mean zero and a unit standard deviation, and by associating W^* with the cutoff rate of return R^* such that $W^*=W_0(1+R^*)$. Since R^* is generally negative, it can be rewritten as $-|R^*|$. In addition, associating the cutoff return R^* with a standard deviate $\alpha > 0$ by setting:

$$-\alpha = \frac{-|R^*| - \mu}{\sigma} \qquad (3)$$

is equivalent to setting:

$$1 - c = \int_{-\infty}^{W^*} f(w)dw = \int_{-\infty}^{-|R^*|} f(r)dr = \int_{-\infty}^{-\alpha} \Phi(\varepsilon)d\varepsilon \qquad (4)$$

where the estimation of VAR is equivalent to finding the deviate α such that the area to the left of it is equal to 1 – c. The deviate α for a chosen certainty level, can be determined by consulting tables of the cumulative standard normal distribution function. Should a confidence level of 95 percent be selected, the value for α is equal to 1.65 below mean 0. To find R^*, the equation for α is rewritten as:

$$R^* = -\alpha\sigma + \mu \qquad (5)$$

and is placed in the equation for VAR below the mean, assuming that μ and σ are expressed on an annual basis (so the time interval considered is Δt, in years):

$$VAR\mu = E(W) - W^* = -W_0(R^* - \mu) = W_0\alpha\sigma\sqrt{\Delta t} \qquad (6)$$

In concordance with this formula, Jorion states that “the VAR figure is simply a multiple of the standard deviation of the distribution, times an adjustment factor that is directly related to the confidence level”.[20] In terms of absolute dollar loss, VAR is expressed as:

$$VAR(zero) = W_0 - W^* = -W_0R^* = W_0(\alpha\sigma\sqrt{\Delta t} - \mu\Delta t) \qquad (7)$$

[20] Ibidem. P. 91

Regulation based on Basle Committee recommendations usually requires to provide a minimum capital requirement based on the estimated VAR multiplied by a safety factor of three. Should the VAR model employed present 10 or more inconsistencies during a 250 trading day testing period, such factor can increase to four.[21]

VAR was originally developed for banks to cope with the market risks entailed by sophisticated financial instruments such as derivatives. As it will be described further on, contingent liabilities such as guarantees can be regarded as derivative instruments similar to put options. The price path of this put options can be estimated stochastically by means of structured Monte Carlo simulations from which a VAR figure can be measured. Since quite often no historical data is available for infrastructure-related contingent liabilities, Monte Carlo simulations are useful to elaborate a distribution of returns drawn from hypothetical random changes in prices based on volatility parameters determined ex-ante based on factors relevant to infrastructure projects, such as GDP growth or traffic flows. This approach will be detailed in section 4.

[21] Scott and Wellons, op cit. P. 277

2. Fiscal Risk and Management of Contingent Liabilities

Traditional public budgeting, based upon cash flows, does not account for the future outlays that can result from contingent liabilities. The true costs for the government resulting from contingent liabilities employed in infrastructure projects might be masked, thus distorting the true fiscal situation a country will be eventually facing. The sudden activation of contingent liabilities due to an unexpected event may put government finances under severe strain, hindering development and even reversing economic gains from investment previously attracted to the country by means of contingent forms of support. If impending liabilities are neither known nor recorded, they cannot be effectively controlled.[22]

Public Management of Contingent Liabilities

Schick (199) suggests among four different approaches to control and manage fiscal risks entailed by contingent liabilities: a) the first one requires a publicly open stance on the part of the government regarding the types of risks it faces, as well as the volume and eventual costs these commitments will represent, and an estimation of the probability that such contingent obligations will be eventually triggered; b) the second approach implies the incorporation of risks assumed by the government in the current budget process, where the more direct and explicit the risk, the greater the suitability of budgeting proper resources to cover for the estimated costs of the existing contingent liabilities; c) in the third approach, the government should limit risks before they are taken by establishing criteria ruling whether or not governments should assume contingent commitments; d) the fourth approach envisages the reliance of governments in market-type mechanisms to either entirely or partially transfer risk, and risk assessment, to the private sector.

In line with Schick's first three points, Lewis and Mody (1997) advocate an integrated risk management approach to be implemented by governments, comprising:

1. compilation, identification and classification of risks confronted;
2. measurement of risk exposure;
3. incorporation of risk exposure figures in national accounts and budget;
4. determination of the government's tolerance to risk and definition of criteria for the establishment of sufficient unexpected loss reserves;
5. implementation of risk exposure supervisory and controlling systems.

As can be perceived, these authors agree that governments should systematically record, assess, budget, and publicly acknowledge the risk exposure resulting from their contingent liability holdings. The next subsections will describe the components of an integrated risk management approach following Lewis and Mody's proposal.

[22] Schick, op cit. P. 5

Identification

At the simplest level, a government should have a clear idea of the obligations faced by its different entities and agencies, and for which it may ultimately be held liable. Along this line, Polackova (199) has elaborated a matrix useful to map and inventory the fiscal risks assumed by governments. It is also helpful to recognize policies that can be implemented to deal with these risks. The fiscal risk matrix distinguishes between four types of fiscal risks:[23]

- *Explicit liabilities*, whose repayments are established by a law or a contract;
- *Implicit liabilities*, which involve a moral or expected obligation on the part of the government that is not mandated by law, but rather based on public expectations, political pressures, or the role of the State as understood by the corresponding societies.
- *Direct liabilities*, which will occur for certain and thus are predictable based on determined factors.
- *Contingent liabilities*, which are obligations activated by a discrete event that may or may not occur; both the probability of its occurrence and the magnitude of the resulting outlays are very complex to estimate; probability and magnitude depend on exogenous factors, such as the occurrence of a given event, and on endogenous factors, like the design of the contract binding the government and the existing regulatory and monitoring frameworks.

The combination of these risks give rise to four cross-sectional categories: Direct Explicit Liabilities, Direct Implicit Liabilities, Contingent Explicit Liabilities, and Contingent Implicit Liabilities. Examples of each are provided in the fiscal risk matrix below:

Liabilities (of fiscal authorities, not central bank)	**Direct** (obligation in any event)	**Contingent** (obligations if a particular event occurs)
Explicit (liability is recognized by law or contract)	o Sovereign debt o Budgetary expenditures considered by law o Budgetary expenditures legally binding in the long term (civil service salaries and pensions)	o State guarantees on obligations issued by subnational governments and public and private sector entities o Umbrella State guarantees for various types of loans (mortgages, student loans, agriculture, small businesses) o State guarantees on interest and exchange rates o State insurance schemes (deposits, minimum returns on pension funds, floods)
Implicit ("moral" or expected obligation due to public expectations or political pressures)	o Future recurrent costs of public investment projects o Social security schemes if not required by law o Future health care financing if not required by law	o Default on nonguaranteed debt issued by subnational governments or public and private sector entities o Cleanup of privatized entities o Bank failure beyond state insurance o Investment failure of nonguaranteed pension fund or social security fund o Central Bank defaults on its obligations (currency defense, balance of payments stability, foreign exchange contracts) o Bailouts following a reversal in private capital outflows o Residual environmental damage, disaster relief, military financing

Source: Polackova

[23] Polackova, op cit. P. 2

Out of these four categories, direct explicit liabilities and direct implicit liabilities can be easily estimated and forecasted. However, that is not the case for the remaining two categories:

Contingent explicit liabilities, or a government's legal commitment to assume obligations conditional on the occurrence of future events, amounts to a concealed subsidy which may distort the market and may unexpectedly place a government's financial position under serious strain.[24] As mentioned before, traditional public budgeting based on cash flows fails to account for the magnitude and probability of the costs that such instruments may represent to governments in the future. In addition, this means that these commitments are not taken into consideration in overall fiscal planning and analysis. As indicated in the matrix, guarantees are among the most common contingent explicit liabilities, more so in the case were risk-sharing is not rigorously specified between the parties, both in terms of financial coverage (partial or total loan coverage) and risk coverage (type of risk hedged).

Contingent implicit liabilities are obligations that are not officially recognized by the government, and thus are not reported in the fiscal expenditure plans. Governments might be forced by public opinion, or interest groups, to assume these liabilities upon the occurrence of a failure in a given sensitive sector of the country. These commitments are the riskier of all given the high uncertainty of the event that may trigger its activation, as well as the difficulty to assess the value at stake, and the extent up to which the government will be willing, or forced, to become involved in bailing out the corresponding sector. In general terms, the weaker the macroeconomic, financial, regulatory and supervising underpinnings, the larger contingent implicit liabilities will be. Additionally, expectations of government intervention in the case of sectorial failure give rise to increased moral risk, encouraging economic agents to behave irresponsibly. On the one hand, it is argued that governments should make this type of liabilities explicit,[25] while simultaneously enacting laws limiting debt issuance by subnational administrations, and extending bankruptcy laws to them. Such actions would clearly signal how the government will respond in the event of contingencies and thus abate moral hazard. On the other hand, and in line with the position held by the IMF,[26] it is held that in order to avoid increasing moral hazard, governments should refrain from making public its intentions regarding contingent implicit liabilities. Conciliating both stances, Schick (199) stresses that no clear cut rule should be applied these liabilities, and that they should be treated in accordance to the particularities of each specific implicit liability.

Upon completion of the fiscal risk matrix, it is possible to easily calculate and report the maximum possible losses implied by each government's obligations. Also, it is possible to identify the particular risk exposures borne by a government.

Measurement of Risk Exposure

Governments should perform, whenever technically possible, a more insightful quantification of the risk exposure entailed by their contingent obligations in addition to determining maximum possible losses.

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[24] Ibidem, P. 5
[25] Schick, op cit. P. 9
[26] Refer to paragraph 67 of the Code of Good Practices on Fiscal Transparency Manual.

In this line, a more useful figure is the likelihood of losses, and thus, the expected losses from contingent liabilities. Determining expected losses allows for resource allocation decisions to be based on real rather than apparent costs and benefits since comparison between cash subsidies, guarantees, and other government support alternatives becomes feasible.[27] It also facilitates the forecasting of budget costs and government outlays, and supports the implementation of risk-adjusted performance measures against which governments can manage their exposure to contingent liabilities.

Estimation of expected losses is not a simple task given the fact that, in general, no historic default data is available for specific infrastructure projects. Nonetheless, a number of techniques are at hand to calculate the expected losses arising from contingent liabilities. There are actuarial or statistical techniques, econometric techniques, and what has become to be known as Contingent Claims Analysis, which incorporates option pricing theory, defaultable bond valuation, and stochastic simulations. These techniques are described in section 3.

Inclusion in National Accounts and Budget

It has already been argued that cash-based budgeting provides fiscal incentives to policy-makers to accumulate contingent guarantees rather than other kinds of government support that may ultimately be less costly. This is explained by the fact that cash-based accounting does not consider guarantees as an outlay, but rather as an inflow resulting from the premia that might have been assigned to the guarantee; as a result, governments are induced to raise risk premia rather than to reduce the probability or the magnitude of the claims guaranteed.[28] The eventual outlays are registered only upon the occurrence of the event triggering the contingent liability, hence the costs are to be borne by future administrations. In other words, cash-based budgeting obstructs government accountability and encourages policy-makers to pursue short-term political objectives while accumulating excessive risks in detriment of future fiscal and macroeconomic stability.

Lewis and Mody (1997), Irwin et al (1997), and Polackova (1999) favor accrual-based accounts and budgets which fully incorporate the present value of expected losses of contingent liabilities. Since under this method guarantees do appear as expenditures affecting the deficit, it requires appropriations in the same year of its issuance, thus erasing biases in favor of providing guarantees over other instruments. Moreover, this method does not have an effect on cash-based estimates of the government's fiscal deficit given that such effect takes place and is recorded only after actual cash outlays from the reserve fund are carried out.[29] A discount rate suggested is the government's cost of funding for maturities similar to that of each discounted project. [30] This approach is currently being implemented in Iceland and New Zealand, whereas Canada, Sweden, Australia, the United Kingdom and the United States are in process of adopting it.

In addition to the employment of more advanced accounting methods in national budgeting to reflect the true costs of contingent liabilities, legislative measures must be taken in order

[27] Irwin et al, op cit. P. 14
[28] Lewis and Mody, op cit. P. 142
[29] Ibidem, P. 143
[30] Ibidem, P. 143

to ensure that the government (and its agencies, entities and subnational governments) is required by law to report and assess its holdings of contingent liabilities. Regulations should also be enacted to provide or reinforce institutional risk exposure evaluation and control capacity, and to establish limits and criteria on contingent liability issuance. New Zealand's Public Finance Act of 1989 (paragraphs 27, 35 and 41) and Fiscal Responsibility Act of 1994 (paragraph 10) are good examples of regulation requiring comprehensive reporting of contingent liabilities at all levels of government. Another relevant example is USA's Federal Credit Reform Act of 1990 where guarantees are registered in a subsidy cost basis, which is defined as "the estimated long-term cost to the government of a direct loan or loan guarantee, calculated on a net present value basis, excluding administrative costs"[31].

Risk Tolerance and Configuration of Reserves

Once officially recognizing in the budget and national accounts the expected losses from contingent liabilities, the next step would be to set reserve funds to cover not only for expected losses, but also for unexpected losses in accordance to each government's overall tolerance for risk. Reserve funds provides governments with flexibility in dealing with the activation of contingent liabilities, yet they also represent a cost of opportunity since the funds held in reserve could otherwise be allocated in other public programs.

The level of reserves is determined by a government's risk preference, which in turn should ideally be tied to the risk preference of the median voter,[32] and by the total amount of expected losses from contingent liabilities. Alternatively, the level of reserves could be determined by the total risk borne by the government, taking into account the variability of expected losses and the "correlation between product returns and the opportunity cost of capital".[33]

Unlike total expected losses (which can be independently estimated and then added up to the expected losses from the rest of the projects), total risk is dependent on the interaction between project-specific risks. In other words, there are diversification effects that should be taken into account: pooling project-specific risks that are imperfectly uncorrelated will almost necessarily reduce the volatility[34] of the expected losses of a government's overall portfolio. Otherwise, overall portfolio risk could be overstated and excessive reserves be set aside.

In case total risk is chosen as the basis for determining the reserve levels, the Value at Risk (VAR) approach described above can be helpful since it incorporates diversification effects. As noted before, the VAR approach is harshly criticized for assuming that return distributions are normally distributed, and for requiring historic data which is often not available in infrastructure projects. In response, variants of the VAR have been developed with the intention of either incorporating the non-linearity of portfolios (Delta-Gamma approach) or overcoming the lack of data in specific projects (Contingent Claims Analysis). According to Lewis and Mody, Delta-Gamma is not quite useful to analyzing the VAR of

[31] Schick, op cit. P. 17
[32] Polackova, op cit. P. 16
[33] Lewis and Mody, op cit. P. 145
[34] Copeland and Weston, (1992). P. 184

government guarantees over extended time periods, whereas Contingent Claims Analysis can put forward a more accurate illustration of a government's long-term risk exposure.

Contingent Claims Analysis: VAR and Monte Carlo Simulations

Contingent Claims Analysis combines the VAR approach with Structured Monte Carlo analysis. Structured Monte Carlo simulations approximate the conduct of contingent obligations using computational calculations to generate random price paths,[35] to simulate a number of different scenarios for the government's portfolio of obligations on a target date. Then, the portfolio's VAR can be directly figured out from the distribution of simulated portfolio values.

According to Jorion (1997), Structured Monte Carlo analysis is the most comprehensive analytical tool to measure financial risks and is by far the most powerful method to compute value at risk since it can potentially account for a very broad range of risks.

In general terms, the Monte Carlo method usually consists in the assumption of a probability distribution for each of the (assumedly random) variables of the stochastic model formulated to explain the price behavior of the contingent liabilities portfolio, as well as in the assumption of existing correlations between each liability. Based on these assumptions, an extensive number of random samples representing the stochastic processes is generated. This method allows both for the calculation, through the samples, of the variables whose price we want to determine (output variables), and for the construction of its probability distributions.

Jorion summarizes the computation of VAR by means of Monte Carlo simulation in the following steps:[36]

1. Select an adequate stochastic process and parameters. Structured Monte Carlo is very sensitive to model risk, thus the specification of the stochastic process is crucial. A Geometric Brownian Motion model with a single random variable, is employed below exclusively for illustrative purposes:

$$dSt=\mu t\ St\ dt + \sigma t\ St\ dz \qquad (1)$$

   where St is the price, and dz is a random variable with normal distribution (0, dt), and μ and σ represent the instantaneous drift and volatility at time t. To generate a series of random variables S_{t+i} over a given time to maturity, dS/S is integrated over a finite interval, thus yielding:

$$\Delta S_t = S_{t-1}(\mu\Delta t + \sigma\varepsilon\sqrt{\Delta t}) \qquad (2)$$

   where ε is a random variable with standard normal distribution (0,1)

2. Generate a sequence of variables ε_1, ε_2, …, ε_n, which are employed to calculate prices for S_{t+1}, S_{t+2}, …, S_{t+n}.

[35] Jorion, op cit. P. 231
[36] Ibidem. P. 239

3. Calculate the value of the asset $F_{t+n} = F_T$ under this particular set of prices at the target horizon.

4. Repeat steps 2 and 3 several thousand times (N), thus obtaining a distribution of values (F_{T1}, …, F_{TN}) from which VAR can be directly observed. At the selected significance level c, the VAR is the portfolio value exceeded in c times N replications.

In the real world, portfolios contain several assets with various sources of risk each. The simulation methodology can be extended to consider N sources of risk and assets, as well as their respective correlations with each other.[37]

Special care should be given to the specification of the stochastic process: "given an accurate contingent claims model and the "true" specification of the process governing changes in the price of the underlying asset", Monte Carlo analysis can be a very effective tool to examine the sensibilities of infrastructure liability exposures to small and large movements in the underlying factors.[38]

Valuation in Colombia

Alongside with the World Bank, the government of Colombia embarked in what Lewis and Mody (1997) consider the likely first time that a sophisticated contingent valuation method was applied to government supported infrastructure projects so as to estimate the fiscal risk exposure of the government, to draw lessons oriented towards ensuring an efficient risk allocation in future projects, and to reduce the frequency and magnitude of activations on guarantees.

The projects assessed initially were:[39] the El Cortijo-El Vino toll road, where construction costs and traffic volumes were guaranteed; a joint venture telecommunications project between Telecom S.A. and Siemens, where the government guaranteed the latter company annual minimum cash returns after the construction phase ended, and a privately sponsored power project to supply a state-owned distribution company, where the government provided guarantees for the power purchase agreement.

Afterwards, the Colombian government evaluated a second set of projects:[40] the second air lane of the El Dorado Airport, where guarantees against changes in environmental regulation, tariff adjustments, and traffic flows were provided; the aqueduct of Monteria, where construction costs, and tariff adjustments due to regulatory changes, were guaranteed; the energy buying contract Paipa IV, where the buying contract is guaranteed; the Bogota-Villavicencio highway and the Bogota-Facatativa-Los Alpes highway, where guarantees were provided on construction costs, minimum traffic flows, and regulatory changes leading to tariff adjustments; and the water treatment plant at Tibitoc, where a guarantee was given on minimum water volume available to the plant.

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[37] Refer to Butler (1999) and Jorion, op cit. for comprehensive details.
[38] Lewis and Mody, op cit. P. 147
[39] Ibidem, P. 1
[40] Valoración de Contingencias de la Muestra de Contratos. Report elaborated for the Colombian Ministry of Finance and Public Credit. May 7, 2000.

In all these cases, stochastic simulations were employed to estimate net expected losses and loss variances deriving from a number of sources of risk that were identified by means of these assessments. Also, scenario analyses were carried out to determine the sensibility of each project's risk to changes in different factors.

These assessments incorporated ingenious specific modeling techniques[41] for each of the contingencies depending upon their likely frequency over the life span of the project, whether they occurred over time periods or phases (construction or operation), and the type of triggering event (inflation, GDP growth, currency fluctuations). Hence, contingencies expected to occur only once over the lifetime of the project (natural disaster) were valued differently from contingencies likely to occur many times over the same time period (traffic flows below defined minimum).

Reserves Management

Once a reserve is set up remains the question of where and how to invest the reserve fund arises. Typically, reserve funds are invested in national Treasury securities in order to ensure their availability. This is the case in the USA, and also in Colombia with the recently proposed changes to Law 448. However, it has been argued that reserve administrators should be allowed to place a certain proportion of the reserve funds in higher yielding instruments such as stocks. The issue that arises against this argument is that funds must have immediate availability in order to cover for contingent losses whose time of activation is unknown. Moreover, this approach would not be advisable should the local infrastructure sector be positively correlated with the overall national stock market: in the event of an economic downturn, the impact on the infrastructure sector could trigger the contingent claims on the government; simultaneously, if reserve funds had been placed in the stock market, their value would very likely experience an unexpected decrease as a consequence of economic conditions. Ultimately, Lewis and Mody point out that "the objective in investing the reserve funds should be to maximize the value of the assets in the fund when the costs to the government increase— that is, to invest the reserve funds in assets that provide the best hedge against the government's cost for a given return".

[41] Valoración de Riesgos y Asignación de Garantías: Metodologías Específicas de Valoración de Contingencias. Report elaborated for the Colombian Ministry of Finance and Public Credit. March 30, 2000.

3. Guarantees and guarantee pricing methods

The most common form of government contingent liability support are guarantees, which in broad terms are commitments to bear a risk[42], or more specifically, "a contractual arrangement under which a third party (the guarantor) agrees to fulfill the financial or other obligations of the guaranteed party (the principal obligor) to another party (the beneficiary) in the case of default by the principal obligor".[43]

Guarantees differ from each other based on the kind of risk that they shield against and also in the proportion of the underlying obligation value that it covers for. Hence, there are guarantees on political and regulatory risks, exchange risks, interest rate risks, default and credit risks, construction cost risks, force majeure risks, and demand risks, among many others. Simultaneously, these guarantees are categorized in full and partial guarantees, depending on the level of support a government considers convenient to provide, and the degree of risk exposure it is willing to bear.

Commonly used guarantees are:[44]

- Partial risk guarantees, which offset risks associated with specific concerns regarding the host government;

- Partial credit guarantees, which back debt payments for certain periods of maturity, or certain proportions of total loan obligations, in case of project's default arising from a variety of risks;

- Full coverage guarantees, which cover against all risks and where the guarantor is committed to fully pay the liabilities of the project to a lender in case the former defaults due to any risk;

- Counterguarantees, where the government undertakes the obligation to compensate an external guarantor for all amounts disbursed due to the activation of the guarantee.

Under cash-based accounting methods, the expected costs that they would eventually represent to the government are zero since they remain hidden, thus generating incentives for the accumulation of risk exposure on the part of the government. Therefore, guarantees can ultimately result in the governments' wealth, and not the private investor's, varying alongside with a risky outcome, thus ultimately becoming a severe burden on a country's fiscal resources in the event of an economic turndown.

Guarantees are instruments devised to facilitate risk sharing between economic agents involved in a project. Sound and rigorous design and monitoring of these instruments can allow such objective to be attained with no excessive costs being shifted to the government while still attracting private investment. For instance, provision of partial guarantees (covering less than 100 percent of the underlying loans) may moderate the

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[42] Irwin et al, op cit. P. 5
[43] Smith, op cit. P. 71
[44] Bubnova (1999), P. 3

moral hazard that the guaranteed party would incur in by aligning his interests with those of the government, rather than if it had otherwise been offered a full guarantee. Limiting guarantees to cover debt payments, and not equity returns, or charging a premia or deductible on the guarantees issued may also encourage the guaranteed party to perform thorough risk assessments and to control its operations. It may also deter the guaranteed party from conducting careless operation of the involved projects, expecting the government to bail it out in the event of a negative situation.

In this context, guarantee pricing is a powerful course of action to reduce the likelihood of moral hazard implied by a non-priced guarantee, decreases investors' temptation to demand for excessive coverage, shifts the cost of the guarantees to the consumers of the services provided by the guaranteed project rather than to the general taxpayers, and allow governments not only to cover downside risk, but also to share the upside potential through the acquisition of warrants on the relevant project.[45]

General approaches to guarantee pricing are described below. The first two ones, actuarial and econometric models, are disadvantageous for infrastructure-related guarantees since there usually is no available historic data on such projects from which to run calculations. The third approach is based on option pricing theory, where a guarantee is regarded as an European put option, and may employ stochastic techniques to overcome the drawback of lacking historic data. The fourth method, defaultable bond analysis, perceives the guarantee as the difference between a risk-free bond and a defaultable bond, and thus employs rating agencies' data or financial market assessments to value the guarantee.

Actuarial and Econometric Approaches[46]

Actuarial methods intend to estimate future loss patterns based on prior loss experience. By employing the loss history of a given or comparable program they approximate an annual expected loss distribution, which is then adjusted to signal current trends in loss frequency and severity of the expected losses. They also aim at indicating shifts in the risk-sharing relationship between the government and the guaranteed party. Assuming that the annual adjusted loss distribution remains stable over time, the distribution can then be employed to estimate the expected and unexpected costs of the program for any given year.

On their part, econometric models seek not only to identify future loss patterns and expected loss distributions of a given or comparable project, but also intend to explain their sensitivity to underlying economic or financial factors in order to forecast non-linear trends in loss patterns. Thus, by predicting future movements of these underlying factors, econometric models may estimate the changes of the loss distributions of the projects over time.

Unfortunately, both of these approaches heavily rely on the availability of historic data on the performance of projects to compute their estimations, hence they are useful only when large amounts of data exists, which is rarely the case in project-financed infrastructure projects. An alternative to surmount this shortcoming is the use of stochastic simulations which can be readily employed in option pricing techniques.

[45] Mody and Patro (1995). P. 16
[46] Lewis and Mody, op cit. P. 135

Structural Models: Option Pricing Theory

By interpreting guarantees as a put option and running option pricing models using either the Black and Scholes formulation or the Binomial Tree approaches, it is possible to price guarantees on infrastructure projects. This approaches are used to appraise the components of a firm's liability combination, and thus are called "structural" because they are entirely based on the capital structure of the firm, in other words, their estimations are dependent on the firm-value sharing relationship between a firm's shareholders and bondholders denoted below[47]:

Value of firm = Debt + Equity

Black and Scholes Formulation

In their seminal paper, Black and Scholes (1973), provided the fundamentals for option pricing employing the risk-free rate, the value of the underlying asset, its instantaneous variance per unit of time and the term of the option.[48] It also established that corporate liabilities can be viewed as combinations of simple option contracts. Whenever a company borrows, it acquires an option because it is not really compelled to repay the debt at maturity: should the total value of the company's assets be less than the amount of the debt, the company will be inclined to default on the payment and the bondholders will keep the company's assets.[49]

Thus, when the firm borrows and defaults on its loan, the lender actually acquires the company, yet the shareholders have the option to buy the firm back by paying off the contracted debt. This generalization of option pricing was later extended by Merton (1974 and 1977) in what became known as Contingent Claim Analysis, and which was later adopted by the Office of Management and Budget of the USA to determine the value of deposit insurance, pension insurance, and mortgage guarantees in order to forecast budget costs.[50]

For illustration purposes, the following example of valuation of a guarantee on demand risk provided to a project financed infrastructure project is offered. Note that continuous compounding is employed, yet discrete compounding may be used also.

In corporate finance, the value of a company is defined as:

$$V = \text{Debt} + \text{Equity} = D + S \qquad (1)$$

Since in the case of a project finance type of infrastructure development, the expected cash flows are the only assets held by the Special Purpose Vehicle, the value of a project is equal to:

$$V = \text{Debt} + \text{Income} = D + I \qquad (2)$$

[47] Cossin and Pirotte (2001), P. 15
[48] Lewis and Mody, op cit , P. 138
[49] Hinojosa (2000). P. 1
[50] Lewis and Mody, op cit. P. 138

Assuming that the government has agreed to cover with its guarantee a minimum level of income, equal to the debt obligations of the project, such Minimum Guaranteed Income (MGI) is equivalent to the project's debt. Thus the value of the project could be interpreted to be:

$$V = MGI + I \qquad (3)$$

Then, following the basic relationship between calls and puts, or put-call parity:

Value of Call + Present Value of Exercise Price = Value of Put + Value of Share *(4)*

the following relationship can be established:

Value of Call + Present Value of Debt = Value of Put + Value of Income *(5)*

Where the call is the guaranteed party's option to buy the project after reimbursing their debt, and the put is the guarantee, equivalent to an option for the guaranteed party to sell the project in case of default.[51] So, by incorporating the concept of Minimum Guaranteed Income (MGI), we obtain:

Value of option to buy + Present Value of MGI = Value of Guarantee + Value of Income
(6)

Parting from this relationship, the call and the put (the option to buy and the guarantee) can then be priced using the formulas developed by Black and Scholes:

$$Call = S_0 N(d_1) - Xe^{-rT} N(d_2) = Income_T N(d_1) - MGIe^{-rT} N(d_2) \qquad (7)$$

and

$$Put = Guarantee = Xe^{-rT} N(-d_2) - S_0 N(-d_1) = MGIe^{-rT} N(-d_2) - Income_T N(-d_1) \qquad (8)$$

Where T is the time of realization, r is the risk-free rate of return and:

$$d_1 = \frac{\ln(S_0 / X) + (r + 1/2\sigma^2)T}{\sigma\sqrt{T}} = \frac{\ln(Income_T / MGI) + (r + 1/2\sigma^2)T}{\sigma\sqrt{T}} \qquad (9)$$

and $d_2 = d_1 - \sigma\sqrt{T}$ (10)

Note that the value of the guarantee is equal to the difference between the present value of the Minimum Guaranteed Income and the actual Income perceived at time T.

If the value of the $Income_T$ is greater than the present value of the MGI, then $N(d_1) = N(d_2) = 1$ and $N(-d_1) = N(-d_2) = 0$, then equations 7 and 8 give:[52]

[51] Cossin and Pirotte, op cit. P. 15
[52] Hull (2000). P. 250

$$\text{Call} = S_0\text{- } X = \text{Income}_T - \text{MGI} \qquad\qquad \text{Put} = 0$$

On the contrary, if the present value of MGI is greater than the Income_T, then $N(d_1) = N(d_2) = 0$ and $N(-d_1) = N(-d_2) = 1$, then equations 7 and 8 give: [53]

$$\text{Call} = 0 \qquad\qquad \text{Put} = X - S_0\text{-} = \text{MGI} - \text{Income}_T$$

Then, according to the put-call parity relationship established in equation 6, the expected value of the call at maturity is:

$$\text{Call} = e^{-rT} E[\text{MAX}(S_T - X, 0)] = e^{-rT} E[\text{MAX}(\text{Income}_T - \text{MGI}, 0)] \qquad (10)$$

and the expected value of the guarantee (put) at maturity is:

$$\text{Guarantee} = e^{-rT} E[\text{MAX}(0, X - S_T)] = e^{-rT} E[\text{MAX}(0, \text{MGI} - \text{Income}_T)] \qquad (11)$$

Also note that there are two variables that are unknown: Income at time T and its volatility σ. Given the nature of infrastructure projects, it is unlikely to have historic data available to determine the project's variance and thus estimate a likely price at time T. What can be done in this cases is to approximate a standard deviation from relevant proxies such as national or regional demand history for the service to be provided (history of traffic, in the case of a road project), history of other countries, income elasticity, or GDP growth, among others, and after calculating the standard deviation, run a Monte Carlo simulation to come up with a distribution of returns whose mean value would be used as the input for Income at time T.

The previous example is consistent with the relationship demonstrated by Merton and Bodie (1992), where a risky (or defaultable) bond is a combination of a risk-free bond and a guarantee:

$$\text{Risky bond} = \text{default free bond} - \text{guarantee}$$

Where the value of a guarantee is equal to:

$$\text{Guarantee} = \text{default free bond} - \text{risky bond}$$

Therefore, the value of a guarantee can be determined in principle by calculating the value of a risk-free bond and that of a defaultable bond. The MGI is analogous to the risk-free bond, whereas the Income_T is comparable to the defaultable bond.

Now, let us assume that the concession on the project has a life of 20 years, and that the guarantee provides coverage during that same period of time. The actual income perceived by the project is known at the end of each year. Then, if the perceived income for that year is less than the Minimum Guaranteed Income, the guarantee on demand is immediately triggered. This means that the guarantee can be activated only at the end of each of the 20 years that the guarantee on the project lasts, This in analogous to having 20 independent guarantees on the project with a life span of one year each. Hence, the

[53] Ibidem. P. 250

value of the 20 year guarantee can be calculated by adding up the expected value of each yearly guarantee:

$$Guarantee_T = \sum_{t=1}^{T} e^{-rt} E[MAX(0, X - S_t)] = \sum_{t=1}^{T} e^{-rt} E[MAX(0, MGI - Income_t)] \qquad (12)$$

Option Pricing Hypothetical Example

Suppose that a private firm is trying to raise money to finance an infrastructure project valued in US$100,000. According to cash flow projections, the company needs to raise $70 million issuing a Fixed Rate Bond in the domestic market with a maturity equal to 10 years. The difference will be financed by issuing stocks. Due to disadvantageous economic and politic conditions in the country where the investment will take place, those funds are very hard to get. In such a situation, the company could get support from the government with a partial credit guarantee covering part of the borrowed amount in the occurrence of default on the part of the company. Such partial credit guarantee could enhance the company's credit position by lowering its spread (at 250 basis points) over risk free securities (US Treasury Bills due on 2010).

The guarantee will pay the first loss that equals 50% in each period of interest and principal of the bond issued. The assumption is that the government only guarantees 50% of debt. The bond is not callable. According to the mentioned assumptions, the following table shows the interest, principal , coupon and amount guaranteed by the government for each year:

Year	Interest	Principal	Coupon Value	Partial Credit Guarantee 70% (Interest & Principal) PCG_t
2001	5,106,500	1,400,000	6,506,500	4,554,550
2002	5,004,370	2,100,000	7,104,370	4,973,059
2003	4,851,175	3,500,000	8,351,175	5,845,823
2004	4,595,850	4,200,000	8,795,850	6,157,095
2005	4,289,460	6,300,000	10,589,460	7,412,622
2006	3,829,875	7,000,000	10,829,875	7,580,913
2007	3,319,225	8,400,000	11,719,225	8,203,458
2008	2,706,445	10,500,000	13,206,445	9,244,512
2009	1,940,470	12,600,000	14,540,470	10,178,329
2010	1,021,300	14,000,000	15,021,300	10,514,910

If the expected return on equity is greater than the cost of capital of the company , net revenues of the project should be able to pay by themselves the debt coupons and dividends for the shareholders (because they financed 30% of the project). In this case the government's guarantee will not be activated. However, if for any given reasons the company should experiment liquidity crisis in one or more years (either because the country faces a recession or whatever other reasons) due to a decrease in net cash flows, thus pushing the bond into default, the guarantee is activated to prevent bond default.

Now assume that the partial credit guarantee is set to cover only 50% of debt interest and principal of the following bond:

Instrument	: Fixed-rate Bond
Guarantee Structure	: 70% Principal and Interest
Amount	: $70 million
Market Sector	: Domestic
Maturity	: 10 years
Spread at Launch	: 285 bp above the US Treasury due 2011

The next table exhibits the net cash flows projection that the company would have shown to the government during the negotiation of the project and of the respective guarantee:

Year	Expected Net Cash Flows NCF_t	Bond Coupon	Net Cash Flow to total liabilities Ratio	Partial Credit Guarantee 50% (Interest & Principal) PCG_t
2001	7,254,748	6,506,500	1.115	4,554,550
2002	7,921,373	7,104,370	1.115	4,973,059
2003	9,311,560	8,351,175	1.115	5,845,823
2004	9,807,373	8,795,850	1.115	6,157,095
2005	11,807,248	10,589,460	1.115	7,412,622
2006	12,075,311	10,829,875	1.115	7,580,913
2007	13,066,936	11,719,225	1.115	8,203,458
2008	14,725,186	13,206,445	1.115	9,244,512
2009	16,212,624	14,540,470	1.115	10,178,329
2010	16,748,750	15,021,300	1.115	10,514,910

Partial Credit Risk Guarantee (PCG) as a Put Option

PCG mechanism has the objective to absorb losses when the net cash flows (RTR) are low.

- When $PCG_t < NCF_t$, the net cash flows obtained by the company is considered very low and therefore the guarantee is activated and the government must compensate the company. Comparatively, the firm has the option to sell the total of its collected cash flows to the government at the "price" PCG_t fixed in the guarantee agreement contract. Evidently, the option will only be convenient for the firm if NCT_t results lower than PCG_t. In financial terms, the government sells to the company a put option with an exercise price equal to PCG_t.

- There will be as many put options as years of guarantee payments (bond maturity). In our example due to the maturity is 10 years and each year has coupons, so the government gives the company a **package of 10 put options**.

Given that put options are susceptible of being modeled and valued with different times of expiration, the worth of the package will be simply the sum of the value of each option it is comprised of.

Valuation of Partial Credit Risk Guarantee

Using $G(NCF, t, PCG) = -NCF_0(-d_1) + PCGe^{-rt}N(d_2)$, we have 10 European put options. Three scenarios are supposed, where each has different assumptions about the volatility of the underlying asset (or assets). Assuming that the volatility can be 3%, 5% or 7% for the Net Cash Flows and the risk-free interest rate for the government is 5%, the value of each option is shown in the table below:

Partial Credit Risk Guarantee Using Black – Scholes formulation
(in $ million)

Year	European put option in period t=0 with σ=4%	European put option in period t=0 with σ=8%	European put option in period t=0 with σ=12%
2001	0	0	0
2002	0	0	671
2003	0	1,054	19,470
2004	0	3,709	40,615
2005	1,039	56,106	190,849
2006	1,492	57,100	199,200
2007	4,232	101,400	287,900
2008	30,352	223,800	479,900
2009	84,974	354,000	657,252
2010	75,340	354,474	657,776
Total Value	**197,429**	**1,151,643**	**2,553,633**

In conclusion, the expected cost of the guarantee to the government (package of 10 put options) should be:

If Volatility is 4% **$197,429 + Operating Cost**
If Volatility is 8% **$ 1,151,643 + Operating Cost**
If Volatility is 12% **$ 2,553,633 + Operating Cost**

Binomial Tree Analysis

As an alternative to the Black and Scholes approach described before, the pricing of an option, in this case a guarantee that is equivalent to a put option, can be performed by constructing a binomial tree, where the different possible paths followed by an underlying asset (Income at time T) over the life span of the option are represented. This approach is based on the findings by Cox, Ross and Rubinstein (1979) under a risk-neutral valuation principle. A very helpful reference on this approach is Merton and Bodie (1992). The illustration provided below is based on Hinojosa (2000):

Considering the case where a project-financed infrastructure project is expected to generate at the end of the year either $180 million under economic good conditions

(V_u=\$180 m) or \$60 million under bad conditions (V_d=60). The debt held by the project is \$100 million and matures at the end of the year. A guarantee is offered to provide a \$100 million Minimum Guaranteed Income to cover the project's debt obligations in case economic conditions do not allow the project to do it itself at the end of the year. The risk-free rate of return is 8%. What would be the present value of the guarantee (G) provided by the government?

As explained in the previous section, the guarantee can be interpreted as a put option giving the company the right to "sell" the project's value and receive the guaranteed amount (or exercise price) of \$100 million. Under the good economic conditions, the guarantee would be worthless (G_u= 0) at the end of the year since the project would be able to meet its obligations; on the downside, the guarantee would be triggered to cover the difference between that year's actual income and the MGI set to cover the debt obligations, so the guarantee would be worth \$40 million:

$$G_d = \text{MGI} - \text{Income}_d = \$100\text{ m} - \$60\text{ m} = \$40\text{ m} \qquad (1)$$

The risk-neutral valuation principle states that any option dependent on an underlying asset can be valued on the assumption that the world is risk neutral. It means that for the purposes of valuing an option, it can be assumed that the expected return from all assets is the risk-free interest rate and that future cash flows can be valued by discounting their expected values at the risk - free interest rate.[54]

The underlying asset (Income_T) can take a value equal to S_u with probability p, or S_d with probability (1-p), where u and d are the up or down movements of the underlying asset.

In a risk-neutral world the underlying asset will yield a return r (risk-free rate) equal to:

$$E(S_T) = pS_u + (1-p)\,S_d = (1+r) \qquad (2)$$

or

$$E(\text{Income}_T) = p\,\text{Income}_u + (1-p)\,\text{Income}_d = (1+r) \qquad (3)$$

where p :

$$p = \frac{(1+r)-d}{u-d} \qquad (4)$$

Hence, "risk-neutral" probability in this example can be estimated as:

$$p = \frac{(1+0.08)-0.6}{1.8-0.6} = 0.4$$

So the value of put option or guarantee is equal to:

$$G = \frac{0.4\times 0 + 0.6\times 40}{1.08} = 22.2$$

[54] Ibidem. P. 205

Note that both the $Income_u$ and $Income_d$ at time T are unknown. As explained earlier, given the nature of infrastructure projects, it is unlikely to have historic data available to determine the project's variance and thus estimate the likely upside and downside values of Income at time T. Again, what can be done in this cases is to approximate a standard deviation from relevant proxies such as national or regional demand history for the service to be provided (history of traffic, in the case of a road project), history of other countries, income elasticity, or GDP growth, among others, and after calculating the standard deviation, run a Monte Carlo simulation to come up with a distribution of returns whose mean value would be used as the input for $Income_u$ and $Income_d$ at time T.

An infrastructure project has a life spanning over several years, so a guarantee is likely to provide coverage during that same period of time. The binomial decision tree can be extended to account for the possibility of the guarantee being triggered on subsequent years. Trigeorgis (1999) and Hull (2000) provide valuable insight in the valuation of multi-period binomial trees.

Defaultable Bond Valuation Methodology

A guarantee can be estimated from the relationship established by Merton and Bodie (1992), where a risky (or defaultable) bond is a composite of a risk-free bond and a full credit guarantee:

Risky bond = default free bond - guarantee

By rearranging the equation we obtain:

Guarantee = default free bond - risky bond

Thus, in principle, the value of a guarantee can be determined by calculating the value of a risk-free bond and that of a defaultable bond: the value of the risk-free security is straightforward; the value of a defaultable can be estimated using reduced-form valuation models. These models typically only require a recovery ratio, the price of an identical maturity risk-free bond, and the probability of default. For example, under this approach the value of a defaultable zero-coupon bond with a credit rating *i* at time t, and that matures at time T, $v_i(t,T)$, is equal to:[55]

$$v_i(t,T) = p_{rf}(t,T)[\Phi + (1-\Phi)q_i(t,T)]$$

Where $P_{rf}(t,T)$ is the value of a default-free zero-coupon bond maturing at time T and whose value is known at time t; Φ is the recovery ratio at time T after default, and $q_i(t,T)$ is the probability of default at time T given that the defaultable bond has a credit rating of *i*.

Recovery ratios (the fraction of the face value recovered at time T in case of default) can be figured out from past recovery ratios of similar bonds. The price of the risk-free can be directly observed in the market. The probability of default can be approximated from

[55] Nandi (1998), P. 27

market assessments of default probability for bonds sharing the same characteristics as those of defaultable bond.[56] Such market assessments are performed on a regular basis by credit-rating agencies (such as Moody's™, Standard and Poor's™, and Duph and Phelps™) that categorize securities according to the perceived credit-risk of the issuer, which can be firms, projects or even countries. It should be emphasized that reduced-form models perceive the probability of default of any given security as an exogenous variable calibrated according to credit-rating agencies data or to financial markets series.[57]

Suppose for instance that a guarantee is provided to an infrastructure project. Consequently, the government commits to give the project a Minimum Guaranteed Income (MGI) in the event of the debt obligations being greater than the project's income at time T. In such case, the MGI can be perceived as the risk-free bond since it is fully backed by the government (assuming the guarantor has no credit-risk), whereas given the project's income uncertainty, it can be interpreted to be the risky bond. The MGI is priced at par or face value. The income is discounted at the cost of capital that reflects the credit-risk assessed by the market, either directly determined by rating agencies, or by using as proxies the credit-risk rating assigned to projects with similar characteristics. Hence, the value of a guarantee, in the event the Income at time T is not sufficient to fulfill the project's obligations, can be estimated as:

$$\text{Guarantee} = \text{MGI} - \text{Income}_T \, e^{-rT}$$

As suggested in the previous sections, since it is unlikely to have historic data available to forecast a distribution of returns for Income_T in infrastructure projects, it is advisable to first approximate a standard deviation from relevant proxies, and then run a Monte Carlo simulation to generate a distribution of returns whose mean value would be used as the input for Income_T.

[56] Idem. See also Mody and Patro, op cit. P. 17
[57] Cossin and Pirotte (2001), P. 85

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